

Задача Зельдовича-Тиле

$$f = \frac{2\pi R k_{xum} \times 2 \int_0^L [A(x)] dx}{2\pi r k_{xum} 2L \times [A_0]} = \frac{\int_0^L [A(x)] dx}{L \times [A_0]} \quad (1)$$

$$D(A) \times \left(\frac{d^2[A(x)]}{dx^2} \right) \times R = 2k_{xum}[A] \quad (2)$$

$$\begin{aligned} \xi &= \frac{[A(x)]}{[A_0]}, & \eta &= \left(\frac{2k_{xum}}{D(A) \times R} \right)^{\frac{1}{2}} x \\ d^2\xi &= \frac{d^2[A(x)]}{[A_0]}, & d\eta &= \left(\frac{2k_{xum}}{D(A) \times R} \right)^{\frac{1}{2}} dx, \\ d\eta^2 &= \left(\frac{2k_{xum}}{D(A) \times R} \right) dx^2 \end{aligned} \quad (3)$$

$$\frac{d^2\xi}{d\eta^2} = \xi \quad (4)$$

$$\psi = \eta(L) = \left(\frac{2k_{xum}}{D(A) \times R} \right)^{\frac{1}{2}} \times L, \quad (5)$$

$$\eta(x) = \frac{\psi}{L} \times x; \quad d\eta = \frac{\psi}{L} dx; \quad dx = \frac{L}{\psi} d\eta$$

$$\xi(\eta) = C_1 e^\eta + C_2 e^{-\eta} \quad (6)$$

$$C_1 = \frac{e^{-\psi}}{e^{-\psi} + e^\psi}, \quad C_2 = \frac{e^\psi}{e^{-\psi} + e^\psi}$$

$$\xi(\eta) = \frac{e^{\eta-\psi}}{e^{-\psi} + e^\psi} + \frac{e^{\psi-\eta}}{e^{-\psi} + e^\psi} = \frac{e^{\eta-\psi} + e^{-(\eta-\psi)}}{e^{-\psi} + e^\psi} \quad (7)$$

$$\xi(\eta) = \frac{e^{\eta-\psi} + e^{-(\eta-\psi)}}{e^{-\psi} + e^\psi} = \frac{ch(\eta-\psi)}{ch(\psi)} \quad (8)$$

$$\begin{aligned} f &= \frac{\int_0^L [A](x) dx}{L \times [A_0]} = \frac{\int_0^\psi [A_0] \frac{L}{\psi} \frac{ch(\eta-\psi)}{ch(\psi)} d\eta}{L \times [A_0]} = \frac{L \times [A_0] \times \int_0^\psi ch(\eta-\psi) d\eta}{L \times [A_0] \times \psi \times ch(\psi)} = \\ &= \frac{sh(\eta-\psi) \Big|_0^\psi}{\psi \times ch(\psi)} = \frac{0 - sh(-\psi)}{\psi \times ch(\psi)} = \frac{sh(\psi)}{\psi \times ch(\psi)} = \frac{th(\psi)}{\psi} \end{aligned} \quad (9)$$

$$\ln k_{\phi\phi} = \ln k_{xum} + \ln \{th(\psi)\} - \ln(\psi) + const \quad (10)$$